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The Effect of Using Hyper-Elastic Theory to Improve the Prediction of SANICLAY Constitutive Model

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Abstract — Most of constitutive models which today are used to predict soft clays behavior are not capable to accurately predict the mechanical behavior of highly over-consolidated clays. A reason behind this deficiency may be attributed to the use of hypo-elastic theories in order to simplify the elastic response of clays. In practice, this phenomenon may lead to unsafe design of infrastructures. In this paper, predictions obtained by a recently proposed clay model are compared with the experimental data and its limitations are discussed. Then it is shown that modifying this constitutive model according to the various hyper-elasticity theories improves the predictive capacity of the model.

Keyword — Constitutive Model, Soft Clays, Hyper-elasticity, SANICLAY.

1. Introduction

The increasing interest in using soft clay constitutive models has led to development of reliable constitutive models in order to simulate the mechanical behavior of these soils [1]. Modified Cam-Clay model (MCC) has a capacity to simulate soft soils behavior compared to the current models. Proposed based on the plasticity theory, MCC has been extensively used in designing of geotechnical structures on soft soils. However, the model overestimates the shear stress of over-consolidated clays in small strains, which can lead to unsafe design of infrastructures. Recently used as an appropriate alternative to MCC model, SANICLAY [1, 2] presents more realistic predictions than MCC. Nevertheless, this model still over predicts shear stress of highly over-consolidated samples.

Fig. 1 compares the experimental data and the simulations of the MCC and the SANICALY models, in terms of effective stress paths for undrained triaxial compression and extension tests on hydrostatically consolidated samples of Lower Cromer Till (LCT) under OCR = 1, 1.5, 2, 4, 10 and 20. It can be seen from Figure 1 that the predictions obtained from the SANICLAY in comparison with MCC are favorable. However, the simulated stress paths in both models are over predicted. Also, for over-consolidated samples, the simulated undrained stress paths retain a constant value of mean effective stress initially, which corresponds to a purely elastic stress-strain response. In this paper after introducing SANICLAY model and various elasticity theories, this model is modified based on hyper-elasticity theories in such a way that can predict soft clays behavior more accurately, especially for highly over-consolidated samples.

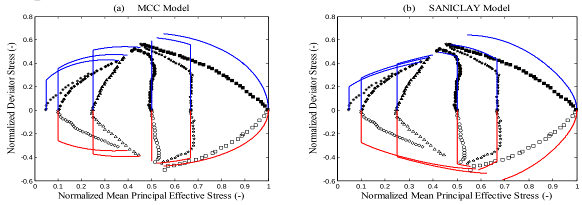

Fig. 1. Comparisons of the MCC and SANICLAY simulation with data for undrained triaxial tests of Lower Cromer Till and various OCR values: predictions by the: (a) MCC model; (b) SANICLAY model

2. General Formulation of Elasto-Plastic Constitutive Model

Each strain increment component is decomposed into elastic and plastic parts:

$$\dot{\varepsilon}_v = \dot{\varepsilon}_v^e + \dot{\varepsilon}_v^p \quad ; \quad \dot{\varepsilon}_q = \dot{\varepsilon}_q^e + \dot{\varepsilon}_q^p \tag{1}$$

where, "ε_v" and "ε_q" are respectively the volumetric and shear strains measured in triaxial space. Superscripts "e" and "p" indicate the elastic and plastic parts of strain rate. When the stress condition is within the yield surface, soil behavior is purely elastic and elastic strains are determined based on elasticity theories. In the next section, elasticity theories used in this study are presented and discussed.

2.1. Hypo-elasticity Theory for Cohesive Soils

In most of the existing constitutive models, due to the simplicity and fewer parameters, the elastic behavior is simulated by a hypo-elastic model. In hypo-elasticity theory, the rate of stress changes is expressed as a function of current stress and strain components. In other words, in this theory, elastic moduli depend on the current stress condition [4]. Although the elastic models belonging to this class are relatively simple, but the principle of the conservation of energy is violated in this class of the elasticity theory. As a result, using hypo-elastic models in numerical analysis may lead to unsafe designing of geotechnical structures [5]. Based on the common version of this theory, the incremental elastic stiffness matrix ([**D**]) is defined as follow:

$$\begin{bmatrix}\dot{p}\\ \dot{q}\end{bmatrix}=\mathbf{D}\begin{bmatrix}\dot{\varepsilon}_v^e\\ \dot{\varepsilon}_q^e\end{bmatrix}=\begin{bmatrix}D_{11} & D_{12}\\ D_{21} & D_{22}\end{bmatrix}\begin{bmatrix}\dot{\varepsilon}_v^e\\ \dot{\varepsilon}_q^e\end{bmatrix}=\frac{p(1+e_{in})}{\kappa}\begin{bmatrix}1 & 0\\ 0 & \frac{9}{2}(\frac{1-2\nu}{1+\nu})\end{bmatrix}\begin{bmatrix}\dot{\varepsilon}_v^e\\ \dot{\varepsilon}_q^e\end{bmatrix} \quad (2)$$

where, " e_{in} " is the initial value of void ratio, " κ " is the slope of unloading-reloading line measured in e-lnp plane, and " ν " is the Poisson's ratio. p and q are respectively the mean principal effective stress, and deviator stress measured in triaxial space. Equation 2 is the simplest possible form of the elastic behavior of cohesive soils.

2.2. Hyper-elasticity Theory for Cohesive Soils

An alternative to hypo-elastic approach is hyper-elasticity, which even for nonlinear pressure-dependent elastic moduli always results in energy conservative behavior. Hyper-elasticity theories are developed based upon the existence of stored energy function. One option to express the stored energy function is using the elastic strain potential function (the Helmholtz free energy function), which is given in triaxial form by $F=F[\varepsilon_v,\varepsilon_q]$. Another option is to express the stored energy potential by the negative complementary elastic energy function (the Gibbs free energy function) $E=E[p,q]$. In hyper-elasticity, stress and strain components are derivates from energy potential functions and by using this theory energy conservation is hold in all stress paths [4]. In this paper, formulations of two hyper-elastic theories for soils are presented and the effects of each theory on improving the prediction of elasto-plastic clay constitutive models are shown.

2.2.1. Hyper-elasticity of Houlsby et al. [4]

Recently, Houlsby et al. [4] introduced a Helmholtz free energy function for clays:

$$\bar{F}=\frac{p_r}{\tilde{\kappa}}.E\,xp(\tilde{\kappa}\varepsilon_v+\frac{3g\varepsilon_q^2}{2}) \quad (3)$$

Where "g" is the material constant and " p_r " is the reference pressure (conveniently taken as 100 kPa). " $\tilde{\kappa}$ " is the elastic compressibility index that produces straight swelling lines in e-lnp space. In this study the relation between " κ " and " $\tilde{\kappa}$ " is established by $\tilde{\kappa}=\zeta\kappa$. Where is the constant multiplier and regarded equal to 1.

Double differentiating of $\bar{F}$ by the strains produces the incremental elastic stiffness matrix ([**D**]) as equation 4:

$$\begin{bmatrix}\dot{p}\\ \dot{q}\end{bmatrix}=\mathbf{D}\begin{bmatrix}\dot{\varepsilon}_v^e\\ \dot{\varepsilon}_q^e\end{bmatrix}=3\tilde{\kappa}gp_r^2\frac{p_0^{2n}}{p_r^{2n}}\begin{bmatrix}\frac{1}{3gp} & \frac{q}{3gp^2}\\ \frac{q}{3gp^2} & \frac{1}{\tilde{\kappa}p}(1+\frac{\tilde{\kappa}q^2}{3gp^2})\end{bmatrix}\begin{bmatrix}\dot{\varepsilon}_v^e\\ \dot{\varepsilon}_q^e\end{bmatrix} \quad (4)$$

Now by considering n=1 and modifying the stiffness matrix of reference model (i.e. SANICLAY that investigated in this study) based on the hyper-elastic theory defined by Houlsby et al. [4] (equation 4), the relation between stress-strain rates can be state as follows:

$$\begin{bmatrix}\dot{p}\\ \dot{q}\end{bmatrix}=\frac{3\bar{G}p}{3\bar{G}\tilde{\kappa}-\underline{n}\eta^2(\frac{p}{p_r})^{1-n}}\begin{bmatrix}1 & n\eta\\ n\eta & 3\tilde{\kappa}\bar{G}p^{n-1}+\bar{n}\eta^2\end{bmatrix}\begin{bmatrix}\dot{\varepsilon}_v^e\\ \dot{\varepsilon}_q^e\end{bmatrix} \quad (5)$$

Where, " $\eta=q/p$ " is the stress ratio. The products of " η " can be identified as a measure of anisotropy, since when it is equal to zero the material becomes isotropic. The fact that the off-diagonal terms depend on the stresses and reduce to zero on the isotropic axis ($\eta=0$) represents what is commonly termed "stress induced cross-anisotropy".

2.3. Flow Rule and Yield Surface

Components of the volumetric and deviatoric plastic strain rates are calculated by normal to the plastic potential function, $g(p,q,\alpha,p_\alpha)$, through:

$$\dot{\varepsilon}_v^p=\langle L\rangle\frac{\partial g(p,q,\alpha,p_\alpha)}{\partial p} \quad ; \quad \dot{\varepsilon}_q^p=\langle L\rangle\frac{\partial g(p,q,\alpha,p_\alpha)}{\partial q} \quad (6)$$

Where, " α " is a parameter defining the orientation of plastic potential function and " p_α " represents the size of plastic potential function. "L" is loading index whose mathematical expression is presented later, and < > are Macaulay brackets. For scalar parameter "x", <x> = x if x>0, and zero otherwise. When stress state is located within the yield function, the predicted response is purely elastic. Plastic strains are generated when the stress state touches the yield function and attempts to step beyond. Herein, yield function, $f(p,q,\beta,p_0)$ may be subjected to both isotropic and rotational hardenings. " β " defines the orientation of the yield function in q-p plane and hence, " β " is a rotational hardening parameter. " p_0 " indicates the extent of the yield function and plays the role of an isotropic hardening parameter. α,β and p_0 evolve as a consequence of the relocation of the yield and plastic potential functions. The general rules describing the evolution of these parameters are:

$$\dot{\alpha}=\langle L\rangle\bar{\alpha} \quad ; \quad \dot{\beta}=\langle L\rangle\bar{\beta} \quad ; \quad \dot{p}_0=\langle L\rangle\bar{p}_0 \quad (7)$$

According to the above evolution laws, loading index *L* controls the pace of the evolution of these hardening parameters. $\bar{\alpha}$, $\bar{\beta}$ and $\bar{p}_0$ are hardening multipliers whose are introduced in the following sections. By imposing the consistency condition, the loading index is calculated by:

$$L=\frac{1}{K_p}\left(\frac{\partial f(p,q,\beta,p_0)}{\partial p}\dot{p}+\frac{\partial f(p,q,\beta,p_0)}{\partial q}\dot{q}\right) \quad (8)$$

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Where, K_p is the plastic hardening modulus:

$$K_p = -\left(\frac{\partial f(p,q,\beta,p_0)}{\partial p_0} \bar{p}_0 + \frac{\partial f(p,q,\beta,p_0)}{\partial \beta} \bar{\beta} \right) \quad (9)$$

3. SANICLAY Model

Dafalias et al. [2] introduced a simple anisotropic clay model within the bounding surface plasticity framework. The model is of non-associated type in which distinct expressions for the plastic potential and yield functions are assumed as eqns. (10) and (11).

$$g(p,q,\alpha,p_\alpha)=(q-\alpha p)^2-(M^2-\alpha^2)(pp_\alpha-p^2)=0 \quad (10)$$

$$f(p,q,\alpha,p_0)=(q-\beta p)^2-(N^2-\beta^2)(pp_0-p^2)=0 \quad (11)$$

Where, "N" and "M" are the model parameters. "M" is the slope of critical state line measured in triaxial q-p plane. In definition of the yield function, "N" plays the same role as that of "M" in plastic potential function. In a general pattern of loading, the yield and plastic potential functions may undergo both isotropic and rotational hardening. The special definitions of the SANICLAY model for hardening multipliers are:

$$\bar{\alpha}=C(\frac{1+e_{in}}{\lambda-\kappa})(\frac{p}{p_0})^2 \left|\frac{\partial g(p,q,\alpha,p_\alpha)}{\partial p}\right| |\eta-\chi\alpha| (\alpha^b-\alpha) \quad (12)$$

$$\bar{\beta}=C(\frac{1+e_{in}}{\lambda-\kappa})(\frac{p}{p_0})^2 \left|\frac{\partial g(p,q,\alpha,p_\alpha)}{\partial p}\right| |\eta-\beta| (\beta^b-\beta) \quad (13)$$

$$\bar{p}_0=(\frac{1+e_{in}}{\lambda-\kappa})p_0 \frac{\partial g(p,q,\alpha,p_\alpha)}{\partial p} \quad (14)$$

Where, "λ" is the slope of the normal compression line in *e*-ln*p* plane. "C" and "χ" are the model parameters. $\alpha^b = M_c$ when $q/p > \chi\alpha$, otherwise $\alpha^b = -M_e$. In this definition, "M_c" and "M_e" are respectively the slopes of the critical state lines measured in the compression and extension modes of triaxial. In the same line, $\beta^b = N$ when $q/p > \beta$, otherwise $\beta^b = -N$. The SANICLAY requires eight parameters. These parameters used in simulations are presented in Table 1.

Table (1) The model parameters used in simulations

Parameter	SANICLAY Parameters			
	M_c	M_e	λ	κ
Value	1.18	0.86	0.063	0.009
Parameter	υ	N	χ	C
Value	0.20	0.91	1.56	16.0

4. The Application of Hyper-elasticity Theory on Improvement the Original Version of SANICLAY Predictions

Base on the triaxial tests, Gens [3] studied the behavior of Lower Cromer Till (LCT) samples in compression and extension modes. LCT is classified as a low-plasticity sandy silty-clay (CL, with liquid limit ω_L=25% and plasticity index I_p=13%), with the main clay minerals being calcite and illite.

Fig. 2 compares the data and the simulations for undrained triaxial compression and extension tests on hydrostatically consolidated samples of LCT for OCR = 1, 1.5, 2, 4, 10 and 20. In particular, Fig. 2 (a, c) makes the comparison in terms of the effective stress paths. Note that the triaxial stress quantities p and q are normalized over $\sigma_{a\text{-}Max}$, (i.e. the maximum axial stress of the preceding consolidation path), and are called "Normalized Mean Principal Effective Stress" and "Normalized Deviator Stress" respectively. However, Fig. 2 (b, d) does the same in terms of the stress-strain response. The ε_a is the axial strain during shearing and called "Value of Axial Strain". As already stated in Fig. 2 (a, b), with respect to using hypo-elastic theory in SANICLAY the simulated behavior by it related to over-consolidated samples in undrained stress paths retain a constant value of p initially, which corresponds to a purely elastic stress-strain response. This type of response continues until the stress point reaches the yield surface, leading to elasto-plastic response until the critical state. Although this model can reasonably predict elastic response of normally consolidated samples and those with small over consolidation ratios (OCR < 2), it cannot correctly simulate the behavior of highly over consolidated samples (OCR>2) and the model predicts a higher shear strength than the observed behavior for these samples which is one of the main drawbacks of this model. In Fig. 2 (c, d), the predictions calculated by Modified SANICLAY model based on hyper-elastic theory (Houlsby et al. [4]), which is found in this study, are depicted versus corresponding experimental data of isotropically consolidated samples. Substantial improvement in the predicted behavior of highly over consolidated samples is observed in Fig. 2(c). Using Hyper-elastic theory instead of hypo-elastic theory is the major reason of this improvement. Also, for low over-consolidated samples (OCR<2) the modified model can predict small inclined stress path when stress condition is within the yield function with an acceptable accuracy. The other important improvement in the predictions of the modified model is more realistic and more accurate estimation of shear strength on the shear stress-axial strain curve (Fig. 2(d)) for over-consolidated samples specially in extension mode.

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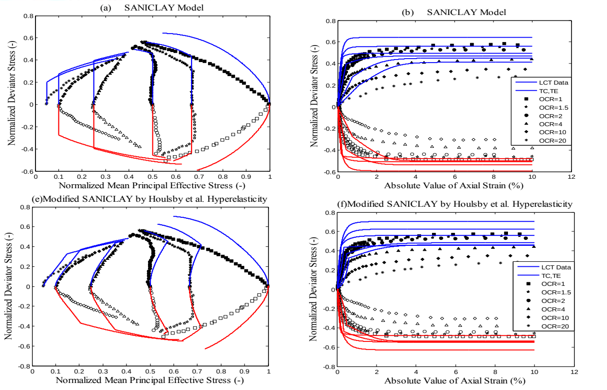

Fig. 2. Comparisons of the simulations with data for isotropically consolidated samples: predictions by (a & b) SANICLAY; (c & d) Modified SANICLAY

In addition, Fig. 3 compares the data and the simulations for undrained triaxial compression and extension tests on K_0-consolidated samples of LCT and OCR = 1, 2, 4 and 7 by SANICLAY (Fig. 3 (a, b)) and Modified SANICLAY (Fig. 3 (c, d)).

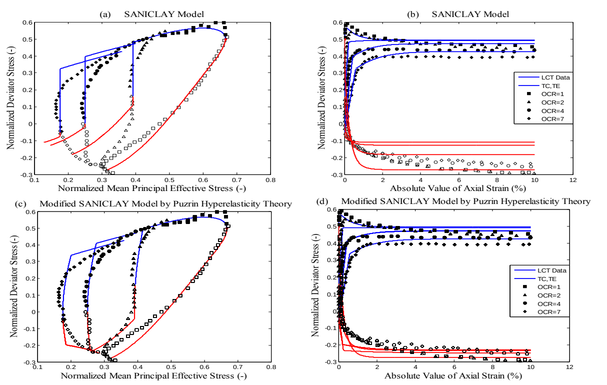

Fig. 3. Comparisons of the model simulations with data for undrained triaxial tests on K0-consolidated (Anisotropically) samples: predictions by (a & b) SANICLAY; (c & d) Modified SANICLAY

As mentioned before, Because of using hyper-elastic theory in the modified model, there is a substantial improvement in the predicted behavior for highly consolidated samples under K0-consolidated and the response is not purely elastic which is shown in Fig. 3(c). In addition, the model which is modified based on Houlsby et al. [4] has reached the ability of predicting the behavior of all of the samples specially in the extension mode corresponding to experimental data, which is shown in the Fig. 3 (c, d).

5. Conclusions

Most of the current constitutive models can predict the behavior of normally consolidated soft soils and those with small over-consolidated ratios (OCR<2) with an acceptable accurate. However, for highly consolidated samples the results are rather imprecise because these models over predict shear strength. In this context, SANICLAY provide better predictions than common models. It does not reach a substantial improvement in highly consolidated samples, however. The major reason is that most of the current models specially SANICLAY use hypo-elastic models to describe the elastic behavior of soils. In this paper, the current version of SANICLAY was modified by two hyper-elastic theory (Houlsby et al. [4]) and was shown that this modification can cause substantial improvement in the soft soils simulated behavior, especially highly consolidated samples.

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