Peak-to-Average Power Ratio Reduction Techniques in OFDM System

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Abstract — According to the demand of advance communication field, there should be high data rate in addition to both power efficiency and lower bit error rate. This demand of high data rate fulfilled by the single carrier modulation with compromising the trade-off between the power efficiency and bit error rate. Again, in the presence of frequency selective fading environment, it is very difficult to achieve high data rate for this single carrier modulation with a lower bit error rate performance. With considering an advance step towards the multi carrier modulation scheme, it is possible to get high data rate in this multipath fading channel without degrading the BER performance. To achieve better performance using multi carrier modulation, we should make the subcarriers to be orthogonal to each other. The Orthogonal Frequency Division Multiplexing (OFDM) technique is its high Peak to Average Power Ratio (PAPR). As we are using the linear power amplifier at the transmitter side so it’s operating point will go to the saturation region due to the high PAPR which leads to in-band distortion and out-band radiation. This can be avoided with increasing the dynamic range of power amplifier, which leads to high cost, and high consumption of power at the base station. This paper presents an efficient technique proposed AMR selective mapping, which is a combination of two kinds of scheme, the implementation method of the Improved SLM OFDM system and Modified SLM scheme with low complexity, which reduces the PAPR. In addition, the analysis of bit error rate performance and the computational complexity for this technique are being discussed here.

Keyword — OFDM, PAPR, BER, SLM

1. INTRODUCTION

Orthogonal frequency division multiplexing has many well-known advantages such as robustness in frequency-selective fading channels, high band width efficiency, efficient implementation, and so on. Orthogonal frequency division multiplexing has made its way into many applications in both wire and wireless environments. Some of well-known examples include digital video broad-casting-terrestrial (DVB-T) digital audio broad casting (DAB). A major drawback of orthogonal frequency division multiplexing at the transmitter is high peak-to-average power ratio of the transmitted signal. This large peak requires linear and consequently inefficient power amplifiers. To avoid operating the power amplifiers with extremely large back-offs, must allow occasional saturation of the power amplifiers. Resulting in band distortion and out of band radiation. Several techniques proposed to reduce peak to average power ratio in Orthogonal Frequency Division Multiplexing system. Selective mapping signal is scrambling technology solutions Selected mapping technique, and interleaving technique improve peak to average power ratio statistics of an orthogonal frequency division multiplexing signal significantly without any in-band distortion or out-of-band radiation. They require side information to be transmitted from transmitter to receiver. In the selective mapping method full group of candidate signal generated on behalf of the same information, as the peak to average transmit power, the basic idea of method of reduction multi carrier modulation system is that the most favorable signal as regards to peak to average power ratio Selection and propagation. A side information about the select the required specifically select candidate with signal transmission. Selected mapping belong to probabilistic class because several candidate signals are generated and one with minimum peak to average power ratio is selected for transmission. It is well known that selected mapping is more advantageous if the amount of side information is limited, but the computational complexity of selected mapping is large. In order to improve the peak to average power ratio reduction performance of selected mapping scheme, we have to increase the number of phase sequences. The computational complexity of selected mapping scheme linearly increases as the number of phase sequences increases, which corresponds to the number of Inverse Fast Fourier Transform required to generate the alternative Orthogonal frequency division multiplexing signals. Even if the selected mapping scheme is simple and distortionless, sometimes its computational complexity is burdensome. In this paper, we propose AMR selected mapping scheme which has...
lower computational complexity with keeping the similar peak to average power ratio reduction performance compared with the conventional selected mapping scheme [3] Selective mapping method is a non-orthogonal frequency distortion algorithms to reduce the peak to average power ratio division multiplexing signal. Selective mapping the concrete realization process in the transmitting end, selective mapping generate U transmitted sequence, Au, U = 1,...., U, independently of one another, through the corresponding algorithm. sends the sequence of the same message, eventually need select the transmitted sequence and average power ratio of minimum peak from sending U the transmitted sequence. At the receiving end, in order to receiving and demodulating signal, to know that if u transmitted sequence. This paper is organized as follows. In Section II, Conventional SLM Scheme. In section III, Modified SLM Scheme with low complexity scheme is introduced. In Section IV, Improved SLM OFDM scheme is introduced. In Section V, AMR SLM OFDM scheme is introduced with Simulation results. Finally, brief concluding remarks are given in section VI.

II. CONVENTIONAL SLM SCHEME

Let A={A_0, A_1, ..., A_{N-1}} denote input symbol sequence in frequency domain, A_k represents complex data of kth subcarrier and N number of subcarriers of Orthogonal Frequency Division Multiplexing signal. T the period of input symbol and NT period of Orthogonal Frequency Division Multiplexing signal. Orthogonal Frequency Division Multiplexing signal is generated by summing all N modulated subcarriers each of that is separated by 1/NT. Then complex Orthogonal Frequency Division Multiplexing signal in time domain is expressed as

$$a_t = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{j2\pi \left(\frac{n}{N}\right)t}, 0 \leq t < NT$$

Where t is a continuous time index .The Orthogonal Frequency Division Multiplexing signal sampled at Nyquist rate can be written as

$$a_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n e^{j\frac{2\pi}{N}k}, k = 0,1,...,N-1$$

Which can also be expressed in the vector form, called an Orthogonal frequency division multiplexing Signal sequence, as a = [a_0, a_1, ..., a_{N-1}]. In fact; corresponds to inverse fast Fourier transform of A.

The peak to average power ratio of Orthogonal Frequency Division Multiplexing signal sequence is defined as the Ratio between its average power and maximum instantaneous power, that written as

$$PAPR(a) = \max_{0 \leq k \leq N-1} \left| a_k \right|^2 / E\left[\left| a_k \right|^2\right]$$

Where E[·] denotes the expectation operator. The conventional selected mapping scheme is one of well-known peak to average power ratio reduction schemes for Orthogonal Frequency Division Multiplexing system, which does not cause in-band distortion or out-of-band radiation. In this scheme, U alternative input symbol sequences A_u, 1≤u≤U, are generated by the component-wise vector Multiplication of the input symbol sequence A and U phase sequences, P_u = [P_{u_0}, P_{u_1}, ..., P_{u_{N-1}}], 1≤u≤U, that is,

$$A_u = [A_{u,0}, A_{u,1}, ..., A_{u,N-1}]$$

$$= A \otimes P_u$$

$$= [A_0 P_{u,0}, A_1 P_{u,1}, ..., AN - 1 P_{u,N-1}], 1 \leq u \leq U$$

(2)

Where ⊗ denotes the component-wise multiplication of two vectors. The phase sequence P_u is generated by using the unit-magnitude complex number, that is P_{u,v} = e^{j\phi_{u,v}}, where \phi_{u,v} ∈ [0, 2\pi]. In general, binary or quaternary elements are used for P_{u,v}, that is {±1}, or {±1,±j}, where j= -1 .IFFT should be performed for each of input symbol sequences to generate alternative OFDM Signal sequences as

$$a_u = IFFT(A_u) = IFFT(A \otimes P_u), 1 \leq u \leq U$$

(3)

Which bear the same input symbol sequence. Then, the Orthogonal Frequency Division Multiplexing Signal sequence a_u with the minimum peak to average power ratio among U alternative Orthogonal Frequency Division Multiplexing signal sequences a_u, 1≤u≤U, is selected and transmitted. Clearly, as U increases, the amount of PAPR reduction for the Orthogonal Frequency Division Multiplexing signal sequence becomes larger. But, for large U, the computational complexity becomes too high mainly due to the U IFFTs.

III. MODIFIED SLM SCHEME WITH LOW COMPLEXITY[11]

In order to achieve large peak to average power ratio reduction in the conventional selected mapping scheme, generate a sufficiently large number of alternative Orthogonal Frequency Division Multiplexing signal sequences, which causes high computational complexity because Inverse Fast Fourier Transform should be performed to generate each alternative Orthogonal Frequency Division Multiplexing signal sequence. Therefore, it is desirable if reduce the number of Inverse Fast Fourier Transform without compromising the peak to average power ratio reduction performance. Let a, and...
where \( a_u \) be the alternative Orthogonal Frequency Division Multiplexing signal sequences, which are generated by the conventional selected mapping scheme as in (3). Using the linear property of Fourier transform, the linear combination of these two sequences can be given as

\[
a_{uk} = c_i IFFT(A \otimes P_i) + c_k IFFT(A \otimes P_k)
= IFFT(A \otimes (c_i P_i + c_k P_k))
\]

where \( c_i \) and \( c_k \) are some complex numbers. If each element of the sequence \( c_i P_i + c_k P_k \) (4) has unit magnitude, \( c_i P_i + c_k P_k \) can also be a phase sequence for the SLM scheme and \( a_{uk} \) can be considered as the corresponding OFDM signal sequence. Therefore, if we have OFDM signal sequences \( a_i \) and \( a_k \), another alternative OFDM signal sequence \( a_{uk} \) can be obtained without doing IFFT. Note that the phase sequence \( c_i P_i + c_k P_k \) is not statistically independent to \( P_i \) and \( P_k \). Now, investigate how to make each element of \( c_i P_i + c_k P_k \) to have unit magnitude under the condition that each element of the phase sequences \( P_i \) and \( P_k \) has unit magnitude. Clearly, the elements of the sequence \( c_i P_i + c_k P_k \) have the unit magnitude if the following conditions are satisfied:

i) Each element of \( P_i \) and \( P_k \) takes the value in \([+1,-1]\);

ii) \( c_i = \pm [1, \sqrt{2}] \) and \( c_k = \pm [1, \sqrt{2}] \).

For two alternative OFDM signal sequences generated from the phase sequences \( \{c_i, P_i + c_k P_k\} \) have the same PAPR, we only consider the case of \( c_i = \pm [1, \sqrt{2}] \) and \( c_k = \pm [1, \sqrt{2}] \).

Since \( |c||2| = |a||2| = 1/2 \), the average power of \( a_{uk} \) is equal to one half of the sum of average powers of \( a_i \) and \( a_k \).

From binary phase sequences, we can obtain \( 2 \times \binom{U}{3} \) additional phase sequences with \( \binom{U}{3} \) \( U \) (U-1)/2 and, thus, total \( U^2 \) phase sequences such as [13]

\[\{P_1, P_2, \ldots, P_U\} = \left\{ \frac{1}{\sqrt{2}}(P_i \pm jP_k), \frac{1}{\sqrt{2}}(P_i \pm jP_k), \ldots, \frac{1}{\sqrt{2}}(P_i \pm jP_k) \right\}\]

A modified SLM scheme can be explained as follows. By combining each pair among \( U \) alternative OFDM signal sequences obtained by using \( U \) binary phase sequences as the above a set \( S \) of \( U^2 \) alternative OFDM signal sequences is generated as

\[
S = \{a_u | u \leq U, a_u \in \mathbb{U}^2\}
= \{a_i | u \leq U, \{\frac{1}{\sqrt{2}}(a_i + j(a_k)), \frac{1}{\sqrt{2}}(a_i - j(a_k))\} \leq k \leq U\}
\]

(5)

Where only \( U \) IFFT's and additional summations of \( U^2 \) pairs of OFDM signal sequences are needed. The computational complexity for the summations of OFDM signal sequences is negligible compared with that of IFFT. Next, we have to select and transmit the alternative OFDM Signal sequence \( a_u \) with the minimum PAPR among the alternative OFDM signal sequences in \( S \), together with the index \( u \). When \( M \)-ary symbols are used, \( \log_2 U \) symbols should be allocated to transmit the side information corresponding to \( u \). This is denoted by \( A_u^{\text{index}} \). A portion of subcarriers of the OFDM signal is assigned for transmission of the index sequence \( A_u^{\text{index}} \), that is, some part of the input symbol sequence \( A \) should be as signed for \( A_u^{\text{index}} \). Thus, the input symbol sequence \( A \) and the alternative OFDM signal sequence \( a_u \) can be split into the data parts \( a_{du} \) and \( a_{du}^{\text{index}} \) the index parts \( A_u^{\text{index}} \) and \( a_{du}^{\text{index}} \) respectively. The alternative OFDM signal sequence with the index signal \( a_{du}^{\text{index}} \) can be written as, for \( 1 \leq u \leq U \),

\[
a_u = IFFT(A_{du} \otimes P_i) + IFFT(A_{du}^{\text{index}}) = a_{du} + a_{du}^{\text{index}}
\]

And for \( U + 1 \leq u \leq U^2 \),

\[
a_u = \frac{1}{\sqrt{2}} IFFT(A_{du} \otimes P_i) + j \frac{b}{\sqrt{2}} IFFT(A_{du}^{\text{index}}) = \frac{1}{\sqrt{2}}(a_{du} + ja_{du}^{\text{index}}) + a_{du}^{\text{index}}
\]

(6)

We may compare the proposed scheme with \( U \) binary phase sequences with the conventional SLM scheme with \( U^2 \) binary phase sequences. These two schemes show the similar PAPR reduction performance for small \( U \). However, as \( U \) increases, the PAPR reduction performance of the proposed scheme becomes worse than that of the conventional SLM scheme with \( U^2 \) binary phase sequences, because \( U^2 \) phase sequences of the proposed scheme are statistically correlated. For example, when \( U=3 \), let the set of three binary phase sequences be given as \( \{P_1, P_2, P_1\} \). In the proposed scheme, the set of nine phase sequences is \( \{P_1, P_2, P_1, P_1, P_2, (1/\sqrt{2})(P_1 \pm jP_2), (1/\sqrt{2})(P_1 \pm jP_3), (1/\sqrt{2})(P_3 \pm jP_3)\} \). The PAPR Reduction performance of the proposed scheme with \( U=3 \) is similar to that of the conventional SLM scheme with as \( U=9 \). Computational Complexity In the proposed scheme, the reduction of the computational complexity comes from the generation of the additional \( U^2 \) of the OFDM signal sequences from \( U \) IFFT's alternative OFDM signal sequences without performing IFFT, whereas the PAPR reduction performance of the proposed scheme with \( U \) Binary phase sequences is similar to that of the conventional SLM scheme with \( U^2 \) binary phase sequences. The complex Multiplications

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and additions are required for IFFT and the complex additions are required for combining the alternative OFDM signal sequences. We consider the computational complexity of the PAPR reduction schemes in terms of complex Multiplication and complex addition. When the number of subcarriers is N = 2^n and U the total Number of IFFTs, the numbers of complex multiplications and Complex additions required for U IFFTs are (N/2) n and NnU, respectively. We also need additional NU2 complex Multiplications for peak power search for U² alternative OFDM Signal sequences. Thus, the total number of complex multiplications is (N/2) Nu + NU². In the proposed scheme, additional N(U²-U) complex additions are needed to generate the additional U²-U alternative OFDM signal sequences. In order to generate U alternative input symbol sequences A_u, we need N(u-1) most significant bit (MSB) inversions, which can be negligible.

The computational complexity reduction ratio (CCRR) of the proposed scheme over the conventional SLM scheme is defined as

$$CCRR = \frac{1 - \text{complexity of the proposed scheme}}{\text{complexity of the conventional SLM}} \times 100\%$$

The CCRR of the proposed scheme over the conventional SLM Scheme with typical values of U and N, which tells us that the proposed scheme becomes computationally more efficient as the N or U increases. Note that the complex multiplication is more complicated than the other operations. When N=2048, the proposed scheme with U=4 can reduce the complex multiplications by 63.5% with keeping the similar PAPR reduction performance compared with the conventional SLM scheme with U=16.

IV. IMPROVED SLM OFDM SYSTEM [12]

In the conventional SLM OFDM system, the signals multiplied by some phase factors selected from the set \{1,-1,j,-j\} before IFFT, and each phase factor is set to have unit magnitude to preserve the power to reduce the implementation complexity. In the improved SLM algorithm, the optional phase factors are various. That is, in a phase sequence with N phase factors, every phase factor is selected from the set \(P_i^\neq = e^{j\frac{2\pi}{N}}\cdot i = 1,2,...,n\) and \(P_i^\neq = \frac{-j}{8}\) (8)

Where n signifies the number of the candidate phase factors in the phase sequence set, and is an integer that isn't more than N. For example, when N is 64, n could be equal to 8. Furthermore, in a phase sequence set with N phase factors, other N-n phase factors may be \(-j/8\). This ensures that candidate phase factors are more than the ordinary SLM scheme. Clearly, the phase sequences have magnitude of 1 and less than 1, which can preserve or reduce the power at the transmitter. In the novel scheme the phase factors corresponding to data sequence X in diverse phase sequences are allocated in random order, although the phase factors are obtained from the same set above, hence, the side information should be the order of the phase factors in the opposite phase sequence, which weights to X and then gains the lowest PAPR. The number of the phase factors is larger than the ordinary SLM scheme. Phase factors is similar to the conventional SLM OFDM system in the structure; therefore the computational complexity is not likely to increase severely. Assume N=64 and n=9 for simplicity. Consequently the set of the phase sequences becomes,

\[P^u = e^{j\frac{\pi}{4}}, j, e^{j\frac{\pi}{8}}, -1, e^{-j\frac{\pi}{8}}, -j, e^{-j\frac{\pi}{4}}, 1, j, ..., -j, \frac{-j}{8}\] (9)

Compared to the ordinary selected mapping the candidate phase factor number becomes large, helps to reduce the peak to average power ratio.

V. AMR SLM SCHEMES

The main idea of AMR method is a combination of the above two kind of scheme, the implementation method of the Improved SLM OFDM system selection range of phase factor by formula (9) to describe the extended interval, combined with the Modified SLM scheme with low complexity. In order to evaluate the performance of the PAPR reduction and the BER with the improved SLM OFDM scheme, the simulation results are given in this section. Fig.1 shows the CCDF of the PAPR of the conventional SLM schemes. Here, we adopt 16-QAM modulation, 64 subcarriers (N=64), 64 IFFT conversion points, and 9 candidate phase factors in a phase sequence set, which means that the number of the distinct phase factors is 9. In Fig.2,3 we suppose an OFDM system with 64 subcarriers and 16-QAM modulation. Compares the CCDF of the PAPR of OFDM signals with the ordinary and the proposed SLM OFDM schemes. In all simulations, 1000 random OFDM data frames are generated to get the CCDF of the PAPR. In fact, when other N-n p_u are equal to \(-j/16\), it can be detected that the PAPR reduction is more, but the BER performance is a little worse. Meanwhile, the power gets larger than \(-j/8\) for p_u while proceeding the inverse conversion at the receiver. In the following three figures of CCDF feature, the PAPR of the original OFDM signal without SLM technique is almost up to 10.5 dB at CCDF=0.1%. In Fig.3 after the conventional SLM scheme is applied, the PAPR are improved to 8.7, 7.6, 6.8 respectively when u is 2, 4, 8. In the proposed SLM and AMR SLM OFDM system, the PAPR are improved to 8.1, 7.1, 6.5, 7.5, 6.1, 5.6 respectively when u is 2, 4, 8. In the modified SLM and AMR SLM OFDM system, the PAPR are improved to 7.9, 6.4, 5.9, 7.5, 6.1, 5.6 respectively when u is 2, 4, 8. We also can see that the PAPR gets smaller when the number u of the candidate phase sequences is larger. It is easy to explain. Along
with u becoming larger, the optional phase sequences to reduce the PAPR are more, that means, the probability of the PAPR reduction gets larger. In Fig.3 and Fig.4, when u is constant and fixed, the PAPR with the proposed SLM scheme is almost 0.6 dB less than that in the ordinary SLM scheme. Besides, we can make out that the novel SLM scheme is appropriate for multiple modulation methods. However, it can be figured out that the PAPR reduction capability decreases as the number of the modulated subcarriers increases. The reason is that the more the number of the subcarriers is, the larger the probability of the same phase for an OFDM symbol is, and the smaller the PAPR reduction is. In this paper, we only consider the channels with additive white Gaussian noise (AWGN). BER performance of the original and proposed SLM scheme with QPSK modulation, N=64 and the random AWGN channel. Due to the noise in the channels, the side information and the OFDM signals are both likely to be distorted, which results in the bad BER performance. BER performance of the proposed SLM scheme is a little worse than that of the conventional SLM scheme, but when the signal noise ratio is larger than 12, the BER performance of the proposed SLM scheme is identical to that of the ordinary SLM system, and the error bit ratios are both equal to 0. In brief, the proposed SLM scheme more apply to the system that the number of the subcarriers is relatively small.
VI. CONCLUSION

We have proposed AMR SLM scheme for the PAPR reduction of OFDM system, which considerably reduces the computational complexity while it maintains the similar PAPR reduction performance compared with the comparable conventional SLM, improved SLM and modified SLM schemes.

ACKNOWLEDGMENT

My warmest gratitude for lecturer Wang Jianping; my supervisor, for her constant help, guidance and encouragement. My thanks goes to my family for great confidence in me all through these years. I thank all my friends and my classmates whom gave me help, guide, encourage and time to listen to me, help me during facing any difficult through my paper.

REFERENCE


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